CSE 431

Lecture 1 Intro to the Theory of Computation

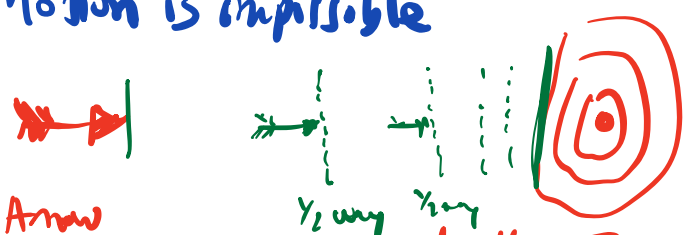
Prehistory of Computation

A lot of the development was inspired need to remove bugs in reasoning

Ancient Greece

Zeno's Paradoxes of Infinity

- Motion is impossible



can never reach the Target

must perform an infinite # of tasks

Can't get to infinity ⊗ contradiction

- Achilles & Tortoise

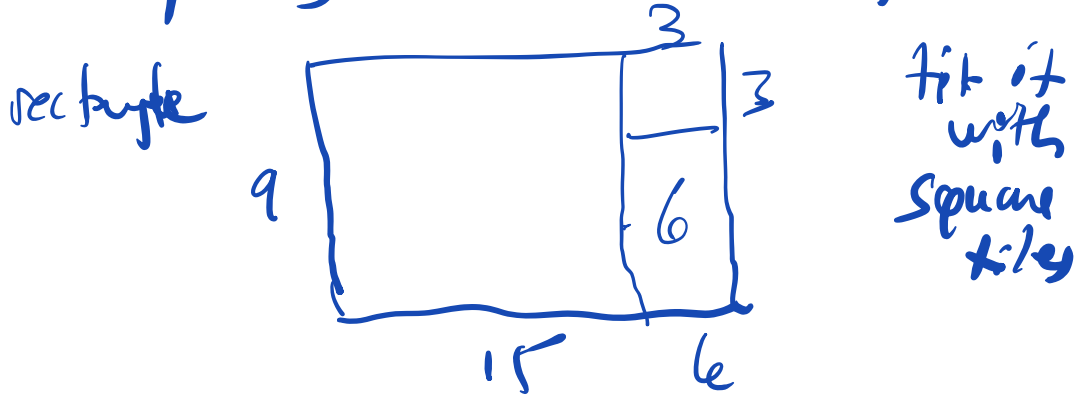
Achilles can never overtake tortoise

Tom Stoppard play Jumpers

Euclid's Elements

- 5 postulates (axioms) of geometry
- Foundation of rigor for millennia

Why GCD? (integers ≡ lengths)



17th and 18th Century Math

- Calculus - Newton & Leibnitz
  - infinitesimals (infinitely small)
  - no good logical foundation
- Fourier: infinite series of trig functions can represent any function

19th Century

Paradoxes of Series

Of eg $\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} \cdots$

$= 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} \cdots - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \cdots\right)$

$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \cdots - 2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \cdots\right)$

$= \underbrace{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}} \quad \underbrace{- \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots\right)}$

$= 0$

Fourier did this kind of thing all the time in his proofs!

Non-Euclidean Geometry (1830)

Each in fundamental parallel postulate ..... doesn't follow from others

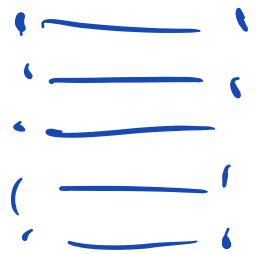
world where it isn't true!

Boole Laws of Thought (1854)

Prop logic

Frege Predicate Logic

Cantor (1873) Cardinality of Real Definite sets



aleph
$\aleph_0$ $\aleph_1$ $\aleph_2$
"nah" → → →

sets are same size
iff 1-1 correspondence

$|\text{natural \#s}| = |\text{rationals}| \neq |\text{reals}|$

Kronecker · "God made integers, the rest is the work of man"

Frege
1893
Foundations of Arithmetic

A = "The set of all infinite sets"
$A \in A$

Peano
1889

Axioms for natural numbers
+, *
induction
$\forall x (P(x) \rightarrow P(x+1))$
$P(0)$
$\therefore \forall x P(x)$

Russell's Paradox (1902)

Using Frege's notation

S = "The set of all sets that are not members of themselves"

Is $S \in S$?

Yes *

No *

Logicomix: An epic search for the truth: graphic novel:

Zermelo-Fraenkel set theory

$\forall x \, (x \notin x)$

ZFC